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A random layer-stripping method for seismic reflectivity inversion

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Abstract. Reflection coefficients and arrival times, together with seismic velocities, are significantly important for possible evaluation of reservoir properties in exploration seismology. Reflectivity inversion is one of the robust inverse techniques used to estimate layer properties by minimising misfit error between seismic data and model. On the other hand, the layer-stripping method produces subsurface images via a top-down procedure so that a given layer is modelled after all the upper layers have been inverted. In this paper, we have combined these two methods to develop a new *random layer-stripping* scheme which first determines the reflectivity series via a random-search algorithm and then estimates P-wave velocities. The first step can be viewed as a variant of sparse spiking deconvolution, and the second step is accomplished by considering empirical relations between density and P-wave velocity. The method has been successfully applied to Marmousi synthetic data to examine dipping reflectors and velocity gradients, and it has been found to be quite reliable for analysing complex structures. A comparison with minimum entropy deconvolution showed that our inversion algorithm gives better results in detecting the amplitudes and arrival times of seismic reflection events.

Key words: layer-stripping, reflectivity series, seismic inversion, sparse spiking deconvolution.

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Introduction

Reflection coefficients and arrival times of seismic events are of significant interest in exploration seismology for their importance in reservoir time-lapse monitoring, AVO analysis, seismic impedance modelling, etc. Deconvolution of the seismic traces is a tool for reflectivity estimation, but the results exhibit a range of accuracy depending on the assumptions made in different implementations. Algorithms which apply deconvolution operators to the recorded seismic trace are sensitive to the phase of the wavelet and to filter length, and results may vary due to weak parameterisation. On the other hand, inversion-based solutions provide more reliable results for the sparse spike deconvolution problem, and can be used to estimate reflectivity series. Velis (2008), and Hondori et al. (2011) used inversion algorithms to extract the least number of spikes in a reflectivity model that could reproduce the seismic trace by convolution with a known seismic wavelet. Although Velis (2008) restricted the number of spikes in the reflectivity model to be known and fixed, Hondori et al. (2011) did not make any assumption about the number of spikes in the reflectivity series. They also showed that their method could be used for arbitrary source signatures including minimum and zero phase wavelets. This could make a robust scheme to process various kind of seismic datasets.

In this paper, we exploit the method of Hondori et al. (2011) to extract reflection coefficients and arrival times of the seismic events by using a random search algorithm. Adaptive simulated annealing (ASA) is used as a global optimisation tool to locate the spikes in the reflectivity series, and linear least-squares inversion has been used to estimate the amplitudes of the spikes. The resulting reflectivity series contains accurate arrival times and amplitudes from layer boundaries in the geological model. In order to achieve high-resolution images of the subsurface, we have used this reflectivity series along with Gardner's equation for P-wave velocity and bulk density (Gardner et al., 1974) to develop a model of P-wave velocities. For this part of the research we followed a layer-stripping strategy (Yagle, 1987; Dewangan and Tsvankin, 2006), in which the layers are modelled in a top-down procedure. In this layer-stripping scheme, a given layer is modelled after all the layers above it have been developed. In our algorithm, however, the reflectivity series results from a completely random scheme, and for this reason we call our method *random layer stripping*. A comparison with minimum entropy deconvolution (Wiggins, 1978) shows that our reflectivity model is more accurate and results have higher reliability. As an example, we demonstrate that our method could handle the complex Marmousi synthetic dataset. Dipping events, and lateral and vertical velocity gradients are sufficiently well identified by the algorithm. We believe that our proposed method can be one solution to the general seismic inversion problem of estimating acoustic impedance profiles.

Theory

Developing a reflectivity model

Using a convolutional model of the earth, any synthetic seismic data can be reproduced by convolving a reflectivity series with a wavelet. A noise component may be added to the data for a more realistic situation. Although the convolution of two signals to reproduce synthetic seismic data is simple, the reverse procedure is challenging because of the limited frequency band that is available. When the seismic record is the only known data in the deconvolution problem, it is difficult to extract the reflection coefficients at interfaces. Moreover, the noise component almost always contaminates data and makes the problem more complex. From a mathematical point of view, the convolutional model can be represented as

$$\mathbf{x}(t) = \mathbf{s}(t) * \mathbf{e}(t) + \mathbf{n}(t), \quad (1)$$

where $\mathbf{x}(t)$ is the recorded trace, $\mathbf{s}(t)$ is the seismic source wavelet, $\mathbf{e}(t)$ is the reflectivity series, and $\mathbf{n}(t)$ is the random noise.

Assuming the Goupillaud model of the earth (Goupillaud, 1961), one could derive an equation in matrix form

$$\mathbf{x} = \mathbf{W}\cdot \mathbf{e} + \mathbf{n}, \quad (2)$$

where $\mathbf{x}$, $\mathbf{e}$, and $\mathbf{n}$ are the seismic trace, reflectivity series and random noise functions, respectively, each consisting of m samples. $\mathbf{W}$ is the $m \times m$ matrix of the convolution kernel whose elements are samples of the seismic wavelet. Equation 3 shows the convolution kernel under the zero-phase wavelet assumption, which results in a symmetric matrix; if one uses a minimum-phase wavelet the matrix $\mathbf{W}$ will be lower triangular.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_m \end{bmatrix} = \begin{bmatrix} w_1 & w_2 & w_3 & \cdots & w_m \\ w_2 & w_1 & w_2 & w_3 & \vdots \\ w_3 & w_2 & w_1 & w_2 & w_3 \\ \vdots & w_3 & w_2 & w_1 & w_2 \\ w_m & \cdots & w_3 & w_2 & w_1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ \vdots \\ e_m \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ \vdots \\ n_m \end{bmatrix} \quad (3)$$

where w_1 is the peak sample of the zero-phase wavelet. To characterise layers adequately, we need to extract the reflectivity series, which is represented by $\mathbf{e}$ in equation 2. Based on inversion theory, we search for the best model that satisfies equation 2 within a given tolerance. To achieve the desired model, adaptive simulated annealing searches the model parameter space, i.e., arrival time of the spikes, randomly and locates the reflectivity spikes one by one. After achieving the optimised set of time lags, the amplitudes of the spikes are calculated to best fit the seismic trace in the linear least-squares sense.

Optimisation algorithm

Adaptive simulated annealing (Ingber, 1996) considers a system in an initial state with energy E_1 , and then perturbs the system so that the energy becomes E_2. If $E_2 \leq E_1$ the system is always allowed to move to the second state, but if $E_2 > E_1$ the system is allowed to move to new state with probability P:

$$P = \exp\left(\frac{-(E_2 - E_1)}{T}\right), \quad (4)$$

where T refers to a parameter like physical temperature which is called the annealing schedule. This statement is known as the Metropolis criterion (Metropolis et al., 1953). Ingber (1996) used an annealing schedule

$$T_k = T_0 \exp\left(-ck^{\frac{1}{D}}\right), \quad (5)$$

where T_k is the annealing schedule, T_0 is a large-enough starting 'temperature', k is the time index of annealing, D is the parameter space dimension, and c is a constant for tuning the algorithm for various problems.

In order to locate layers randomly, we define an inverse problem which aims to find the time lags and amplitudes of spikes in the reflectivity series. Distribution of the seismic reflections in a recorded trace is quite random and, as a consequence, we can search for reflection coefficients by using a random-search algorithm. For this purpose, we use the residual 'energy' of the difference between the recorded seismic data and the developing model as a cost function

$$F = \|\mathbf{x} - \mathbf{We}\|^2, \quad (6)$$

where F represents the cost value for each set of data and model. For normally distributed noise with mean zero and standard deviation σ, the expected misfit is mσ^2, where m is the number of samples. So the expected misfit for the model can be calculated (Velis, 2008).

P-wave velocity model

Various methodologies try to estimate the P-wave velocity field from seismic data, mostly based on waveform inversion techniques. Here we use an empirical relation between bulk density and P-wave velocity known as Gardner's equation (Gardner et al., 1974) to develop a P-wave velocity field from the seismic reflectivity series. Gardner et al. (1974) derived a systematic relation between bulk density and P-wave velocity for sedimentary rocks

$$\rho = 1.74 V_p^{0.25}, \quad (7)$$

where ρ and V_p are bulk density and P-wave velocity in t m^{-3} and km s^{-1}, respectively. Peterson et al. (1955) showed that the reflection coefficient of seismic waves with unit amplitude and perpendicular incidence, while travelling from layer 1 to layer 2, can be represented by

$$R = \frac{1}{2}\ln\left(\frac{\rho_2 V_2}{\rho_1 V_1}\right) \quad (8)$$

Substituting Gardner's equation in equation 8 provides a direct equation for extracting V_p from reflection coefficients. We have used this relation to develop P-wave velocity field from reflectivity series, and the results for Marmousi synthetic seismic data will be presented in a later section. The only parameter that is needed is the velocity of the first layer at the top of the model, which can be measured easily by methods like shallow seismic refraction analysis, or obtained from any other a priori information.

Methodology

Spikes are to be located one by one in the following way. The first spike, whose tentative amplitude is equal to the maximum absolute amplitude of the trace but which has an unknown time lag, is convolved with a known seismic wavelet. The convolved trace is then subtracted from the recorded seismic trace to produce a residual trace. The energy of the residual forms the cost value for the optimisation, as equation 6 shows. The cost function is minimised by adjusting the time lag, using adaptive simulated annealing as the search procedure to yield the best arrival time for the spike. This spike, with tentative amplitude but exact time lag, is then saved in the developing model of the reflectivity series. The convolution of the seismic wavelet with this reflectivity series gives a model for seismic trace, which is subtracted from the original recorded trace to produce a new dataset to search for the next spike. This procedure continues, locating the spikes one by one, until the energy of the residual reaches a predefined minimum criterion, i.e. normally the background noise energy, when the search scheme stops automatically. After achieving the best collection of time lags, the amplitudes of all spikes are calculated and finalised by using linear least-squares fitting to the original recorded trace. Figure 1 shows a flowchart of the algorithm that can be used to develop reflectivity and P-wave velocity models.

The signal to noise ratio controls the automatic procedure to switch off the minimisation algorithm automatically. Therefore, there is no need to make any predefined assumption about the number of spikes. The resulting reflectivity series can be compared with the output of sparse spike deconvolution algorithms. However, conventional deconvolution operators are identical to a mathematical inverse of the seismic wavelet, and in those methods the wavelet must be minimum-phase in order to design stable operators. In our method the reflectivity series is constructed without computing any operator, and the result is not sensitive to the phase of the seismic wavelet. When we have achieved the optimised reflectivity series, P-wave velocities can be estimated by using Gardner's equation as described in previous section.

Synthetic data results

First we check how accurately the algorithm will find the interfaces in a seismic model. The simplest case, to locate a single reflection or a seismic wavelet, has been analysed for both minimum and zero phase data. We have selected a Berlage wavelet (Aldridge, 1990) as the minimum-phase seismic wavelet, and the zero-phase wavelet is a Ricker wavelet. Both wavelets have a dominant frequency of 30 Hz but are delayed by 50 and 100 time samples, respectively. As Figure 2 shows, extracting reflectivity spikes by means of ASA does locate arrival times for minimum and zero-phase wavelets precisely, and the amplitudes are accurate. Figure 3 shows a synthetic seismic trace, the reflection coefficient sequence that resulted from ASA reflectivity reconstruction, and the model seismic trace developed from the reflection coefficient series. Here, the signal to noise ratio is 10, and the random noise is normally distributed with zero mean and standard deviation of 0.02. We used a zero-phase Ricker wavelet with dominant frequency of 30 Hz as the seismic wavelet to create the synthetic trace. As is obvious in Figure 3, this method not only recovers the true reflectivity, but also removes random noise from the original trace. The correlation coefficient between the input data and the resulting model is 0.9583. The final residual after subtracting the developed model from original data is mostly the added noise, so this method can remove noise from the data stream as a by-product.

Another synthetic example (Figure 4) shows the ability of the method to detect closely spaced interfaces with either normal or reverse polarity. All the interfaces are recovered in the model and the results are satisfactory. The correlation coefficient between model and data ranges between 0.9492 and 0.9653 which shows that the results are very close to the original data. These examples show that the resulting reflectivity series is accurate and can be used to develop P-wave velocity models.

Marmousi synthetic dataset

In order to process more complex data we have applied the method to the Marmousi synthetic dataset (Versteeg, 1993).

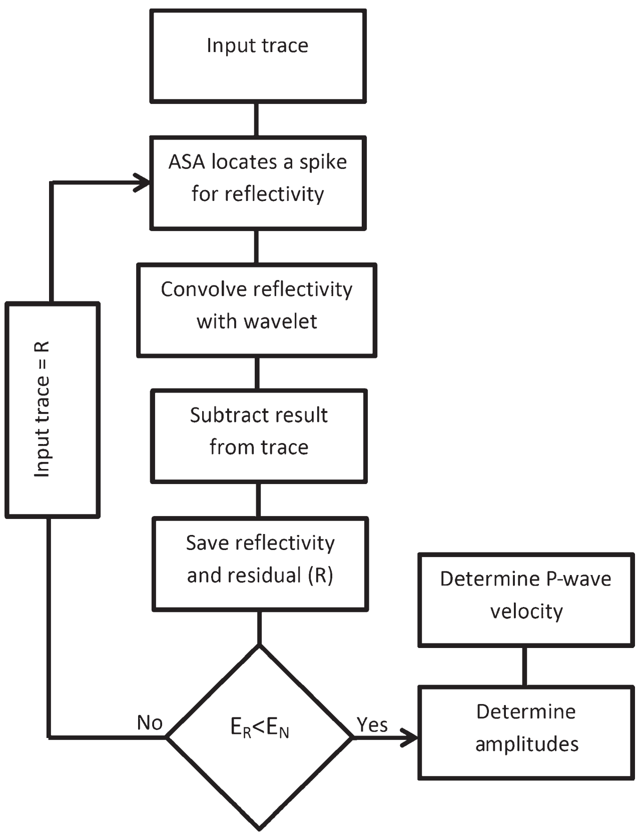

Fig. 1. Flowchart of random layer-stripping algorithm.

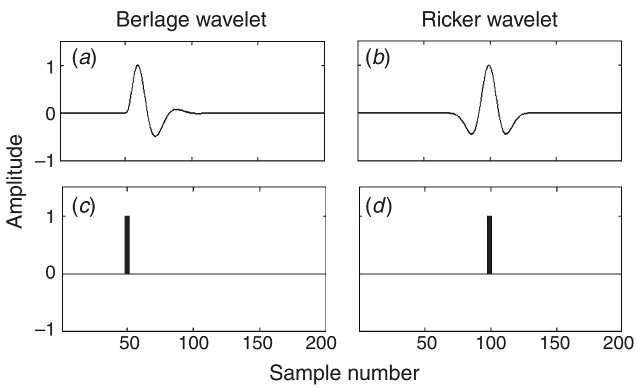

Fig. 2. Extracting reflection coefficients for the minimum-phase and zero-phase wavelets. Top: the wavelets; and bottom: the extracted reflectivity spikes.

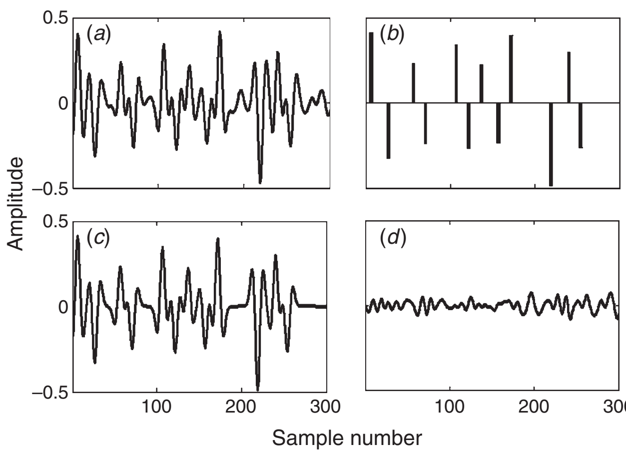

Fig. 3. (*a*) Synthetic input trace; (*b*) extracted reflectivity; (*c*) modelled trace based on the resulting reflection coefficients; and (*d*) the residual after subtracting the final seismic model from original input trace.

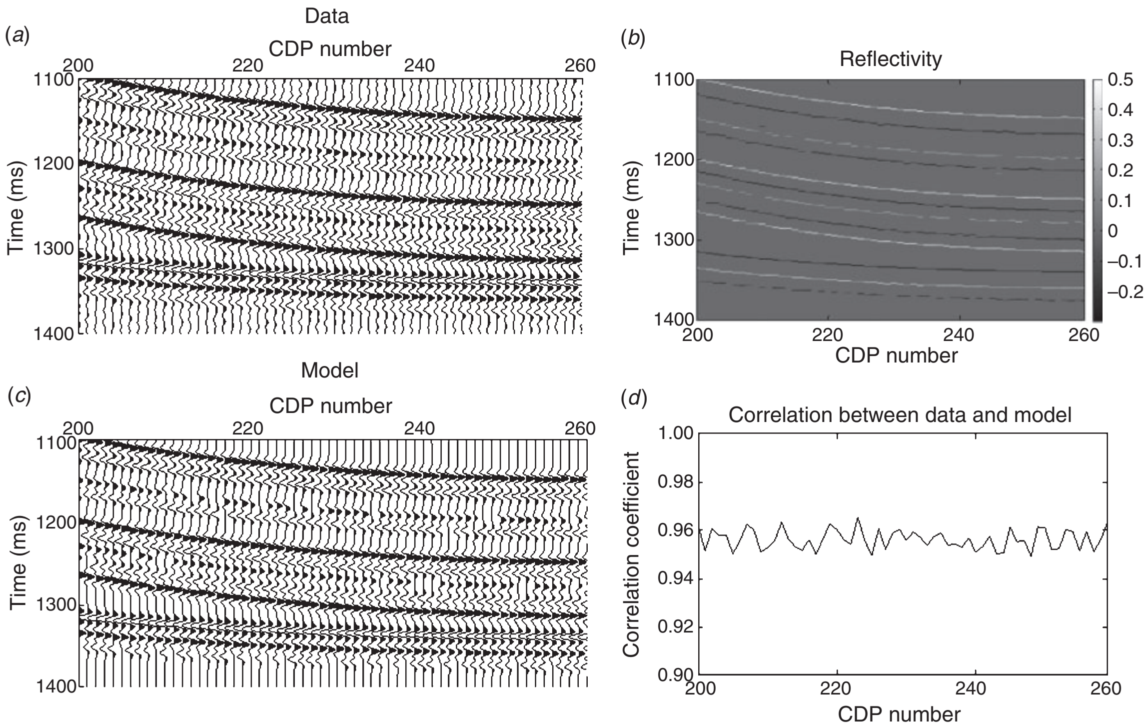

Fig. 4. (*a*) Closely compacted layers as input data; (*b*) extracted reflectivity for the layers; (*c*) the developed model based on reflectivity solution; and (*d*) correlation coefficient between seismic data and developed model.

First we extracted the reflectivity series and then we tried to develop the P-wave velocity model. Figure 5*a* shows the Marmousi synthetic seismic data that was used as input to the algorithm for interface extraction. As Figure 5*b* shows, the interfaces are represented by the reflectivity model very well, and even complex regions have been recognised by the inversion algorithm. Convolution of the reflectivity series with the known seismic wavelet, which has been used to produce the synthetic section, yields the seismic model in Figure 5*c*. This model has a high correlation with the Marmousi seismic data. Figure 6*a* shows the true velocity field of the Marmousi dataset and Figure 6*b* represents the P-wave velocity model resulting from our method. Although the reflectivity series and arrival times have been estimated with high accuracy, it seems that the velocities are, to some extent, different from the actual values. However, as Figure 6*b* shows, the subsurface structure and velocity contrasts

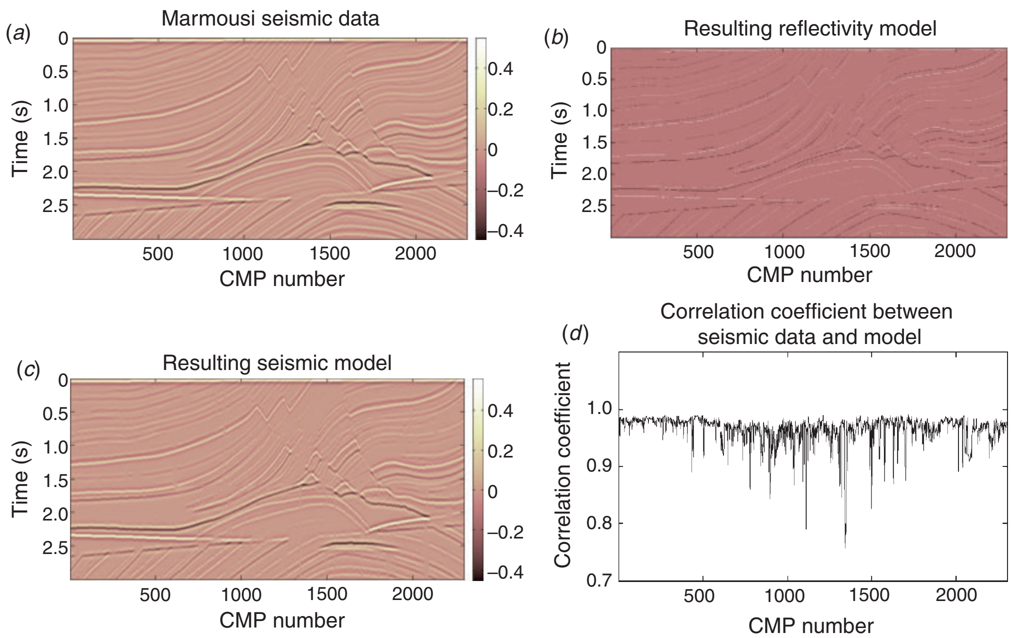

Fig. 5. (*a*) Marmousi synthetic seismic dataset; (*b*) reflectivity series extracted from the Marmousi dataset; (*c*) seismic model reproduced by using the resulting reflectivity series and known seismic wavelet; and (*d*) correlation coefficients between the Marmousi seismic dataset and developed seismic model.

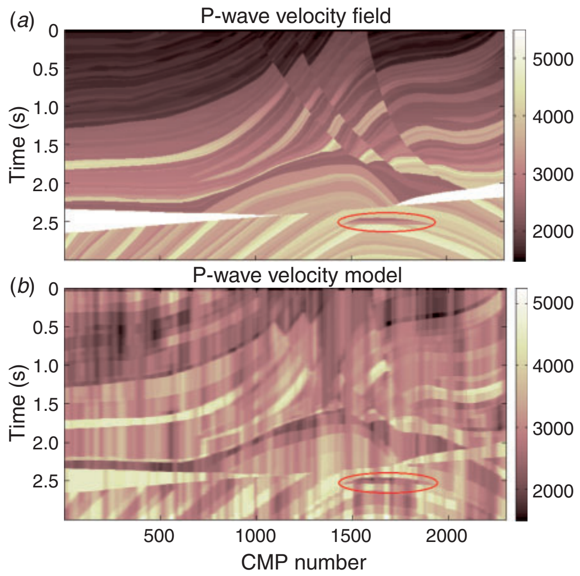

Fig. 6. (*a*) True velocity field of the Marmousi dataset; (*b*) resulting P-wave velocity model. The red oval (in the online version) shows velocity contrasts that were detected in the model.

have been detected in the model. For example, the area highlighted with the red oval in Figure 6*b* shows a velocity contrast zone which has been detected by the method very well.

Field data results

To evaluate the efficiency of our method with real data, we used a section of seismic reflection field data. A time window of post-stack seismic section has been selected and processed. Figure 7 displays the real data results. As these results show, this method can handle real seismic sections with acceptable accuracy. ASA has located the spikes very well and their amplitudes have been calculated with acceptable accuracy. High correlation values between the input seismic section and the resulting model show that the modelled section, based on the reflectivity series recovered by our method, fits the original data and the uncertainty of the solution is slight. Although the seismic wavelet is assumed to be known, we simply used an approximation of the wavelet for this real data, and we found that we could achieve good results even in the case of insufficient information about the seismic wavelet.

Comparison with minimum entropy deconvolution

Minimum entropy deconvolution (MED) was developed by Wiggins (1978) to enhance the resolution in seismic data when high amplitude reflections appear in the trace (e.g. bright spots). MED does not rely on assumptions about the phase of the wavelet or the reflectivity series spectrum. Moreover it tries to find the minimum number of spikes needed to represent the reflectivity just by using the recorded seismic trace. The details of the minimum entropy deconvolution process can be found in Wiggins (1978), Cabrelli (1985), and Sacchi et al. (1994). This method aims to maximise a norm V known as Varimax, which can represent some measure of simplicity in the data. The word 'Varimax' comes from maximising the normalised variance and can be represented in mathematical form as below:

$$V = \sum_i V_i \quad \text{and} \quad V_i = \sum_i \frac{{y_{ij}}^4}{\left(\sum_j {y_{ij}}^2\right)}, \tag{9}$$

where y_{ij} is a matrix containing seismic traces filtered by the MED filter:

$$y_{ij} = \sum_{k=1}^{N_f} f_k x_{i,j-k}, \tag{10}$$

where **x** and **f** are seismic traces and the MED filter, respectively. In order to calculate the MED filter, one can differentiate V with

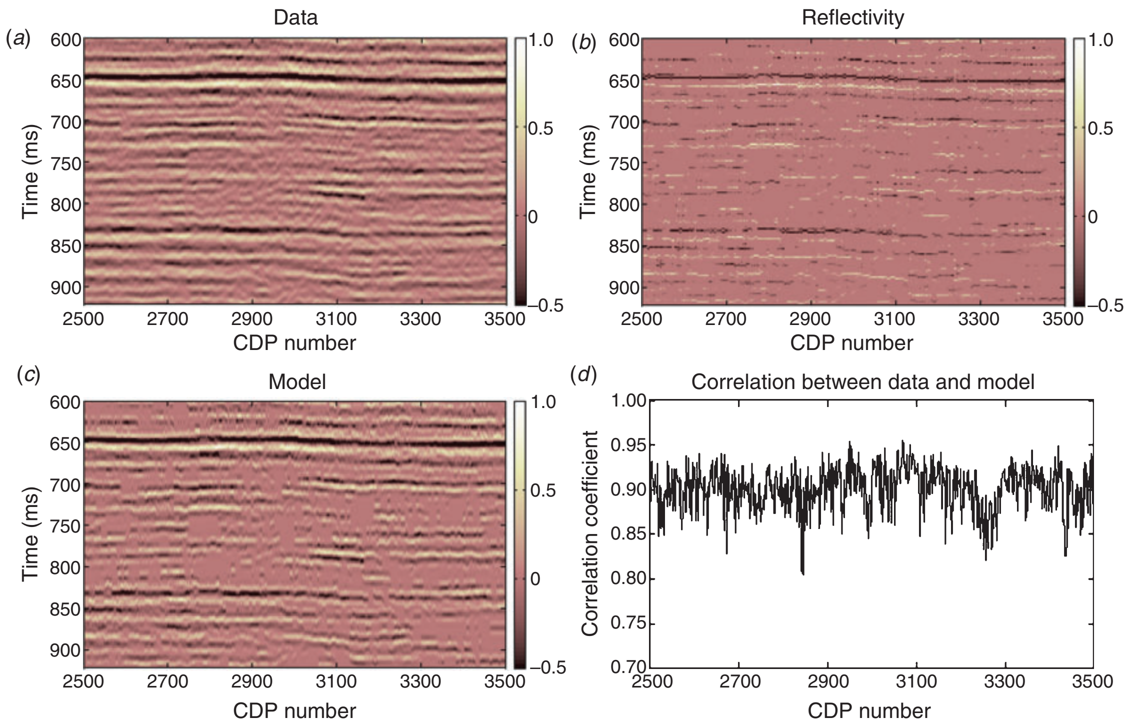

Fig. 7. (*a*) Real seismic section; (*b*) reflectivity series of the earth; (*c*) resulting model of the earth; and (*d*) correlation coefficients between data and model.

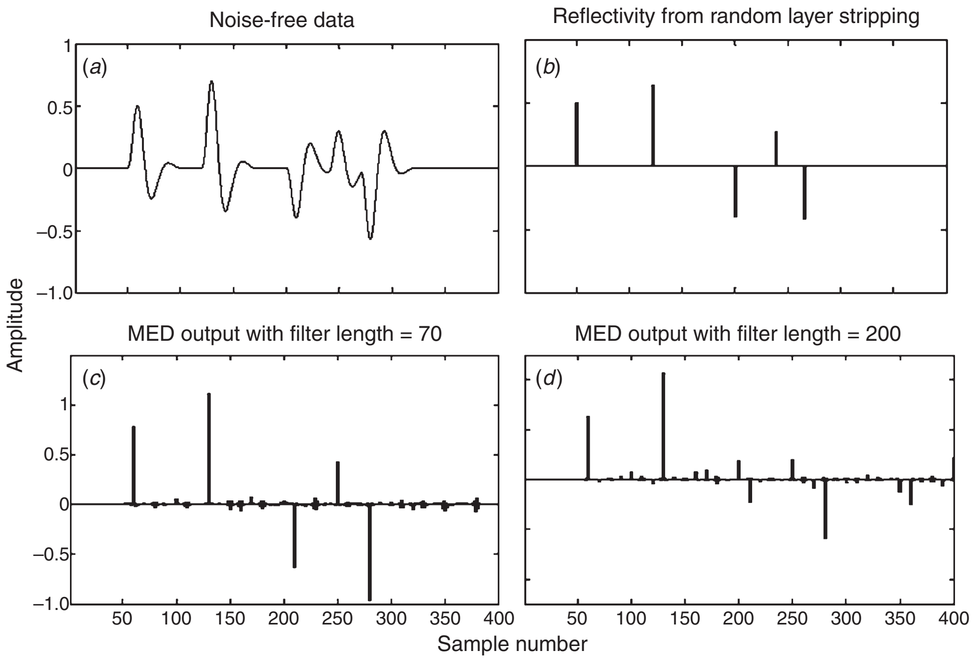

Fig. 8. (*a*) Synthetic seismic trace with five reflectors; (*b*) ASA result for reflectivity series; (*c*) MED results with filter length equal to 70 samples; and (*d*) MED results with filter length equal to 200 samples.

respect to the filter coefficients to maximise the Varimax (Wiggins, 1978).

Here we make a simple comparison between the ASA random layer-stripping method and the MED method. Since these methods have much in common this comparison makes sense. Both methods make no restrictive assumption over the seismic wavelet or the reflectivity series. Also they try to extract reflectivity series by optimising some norm. ASA minimises the l_2 norm of the difference between model and data to locate spikes, while the MED tries to maximise the Varimax. Furthermore, they represent models for the reflectivity of the earth that contain the least number of spikes that can reproduce the seismic trace. By using the MED method one tries to compress the wavelet to a spike. Since the Varimax is unaffected by the spacing or polarity of the spikes (Wiggins, 1978) the MED method requires specific parameterisation to locate the time lags of spikes. Also, the coefficients of the MED filter must be scaled accurately to achieve meaningful amplitudes. Figure 8 shows a synthetic trace which has been processed by ASA and MED to extract reflectivity series. In MED some spikes with small amplitude appear in the output, which is not desirable. Furthermore, the results of the MED process are highly dependent on the filter length. It is obvious in Figure 8 that inappropriate filter length can lead to unrealistic spikes that weaken the accuracy. On the other hand, ASA does detect the time locations of the spikes successfully, and there is no implausible spike in the resulting reflectivity series. Amplitudes are also well developed and represent the true reflection coefficients that had been used in the original synthetic trace. Therefore, the results from ASA reflectivity modelling are more accurate, and this method performs better than MED algorithms.

Conclusions

We have developed a random layer-stripping method to recover reflectivity series and the P-wave velocity field, by combining reflectivity inversion and layer-stripping algorithms. Adaptive simulated annealing has been used as a global optimisation tool to locate spikes in the reflectivity model, in a random search scheme, and linear least-squares fitting determines the amplitudes of the reflections. By including Gardner's equation in the reflection coefficient expression, we could develop a method to estimate the P-wave velocity field in a top-down scheme for the Marmousi synthetic dataset. Our method was efficient in detecting the series of interfaces in both synthetic and real seismic data. Moreover, the method detects velocity anomalies and dipping reflectors very well. The resulting reflectivity series from our method is a solution to the seismic sparse-spike deconvolution problem. Comparison with the minimum entropy deconvolution method showed that our algorithm would perform better in the detection of reflectivity spikes.

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